Chapter 14: Equations

Equation 14.1:

$$y_i = a + \sum_{l=1}^{k} b_l x_{li} + e_i$$

Equation 14.2:

$$\hat{y}_i = a + \sum_{l=1}^{k} b_l x_{li}$$

Equation 14.3:

$$0 = \sum_{i=1}^{n} e_i$$

Equation 14.4:

$$0 = \sum_{i=1}^{n} e_i x_{pi} = \text{COV}\left(e_i, x_{pi}\right) \text{ for all } p = 1, \ldots, k$$

Equation 14.5:

$$a = \overline{y} - \sum_{l=1}^{k} b_l \overline{x}_l$$

Equation 14.6:

$$x_{pi} = c + \sum_{l=1}^{p-1} d_l x_{li} + \sum_{l=p+1}^{k} d_l x_{li} + e_{\left(x_p, x_1 \ldots x_{p-1} x_{p+1} \ldots x_k\right)i}$$

Equation 14.7:

$$y_i = g + \sum_{l=1}^{p-1} h_l x_{li} + \sum_{l=p+1}^{k} h_l x_{li} + e_{\left(y, x_1 \ldots x_{p-1} x_{p+1} \ldots x_k\right) i}$$

Equation 14.8:

$$b_p = \frac{\sum_{i=1}^{n} e_{\left(x_p, x_1 \ldots x_{p-1} x_{p+1} \ldots x_k\right) i} e_{\left(y, x_1 \ldots x_{p-1} x_{p+1} \ldots x_k\right) i}}{\sum_{i=1}^{n} e^2_{\left(x_p, x_1 \ldots x_{p-1} x_{p+1} \ldots x_k\right) i}}$$

$$= \frac{\sum_{i=1}^{n} \left(e_{\left(x_p, x_1 \ldots x_{p-1} x_{p+1} \ldots x_k\right) i} - \bar{e}_{\left(x_p, x_1 \ldots x_{p-1} x_{p+1} \ldots x_k\right)} \right) \left(e_{\left(y, x_1 \ldots x_{p-1} x_{p+1} \ldots x_k\right) i} - \bar{e}_{\left(y, x_1 \ldots x_{p-1} x_{p+1} \ldots x_k\right)} \right)}{\sum_{i=1}^{n} \left(e_{\left(x_p, x_1 \ldots x_{p-1} x_{p+1} \ldots x_k\right) i} - \bar{e}_{\left(x_p, x_1 \ldots x_{p-1} x_{p+1} \ldots x_k\right)} \right)^2}$$

Equation 14.9:

$$R^2 = \text{COV}\left(y_i, \hat{y}_i\right)$$

Equation 14.10:

$$\text{adjusted } R^2 = 1 - \frac{\left(\frac{\sum_{i=1}^{n} e_i^2}{n-k-1} \right)}{\left(\frac{\sum_{i=1}^{n} \left(y_i - \bar{y}\right)^2}{n-1} \right)} = 1 - \frac{\sum_{i=1}^{n} e_i^2}{\sum_{i=1}^{n} \left(y_i - \bar{y}\right)^2} \left(\frac{n-1}{n-k-1} \right)$$

Equation 14.11:

$$y_i = \alpha + \sum_{l=1}^{k} \beta_l x_{li} + \varepsilon_i$$

Equation 14.12:

$$\mathrm{E}(a) = \alpha \text{ and } \mathrm{E}(b_p) = \beta_p \text{ for all } p = 1, \ldots, k$$

Equation 14.13:

$$\mathrm{E}(b_p) = \beta_p + \beta_k \frac{\sum_{i=1}^{n} e_{(x_p, x_1 \ldots x_{p-1} x_{p+1} \ldots x_{k-1})i} e_{(x_k, x_1 \ldots x_{p-1} x_{p+1} \ldots x_{k-1})i}}{\sum_{i=1}^{n} e^2_{(x_p, x_1 \ldots x_{p-1} x_{p+1} \ldots x_{k-1})i}}$$

Equation 14.14:

$$\mathrm{E}(b_p) = \beta_p + \sum_{m=k-q}^{k} \left[\beta_m \frac{\sum_{i=1}^{n} e_{(x_p, x_1 \ldots x_{p-1} x_{p+1} \ldots x_{k-q-1})i} e_{(x_m, x_1 \ldots x_{p-1} x_{p+1} \ldots x_{k-q-1})i}}{\sum_{i=1}^{n} e^2_{(x_p, x_1 \ldots x_{p-1} x_{p+1} \ldots x_{k-q-1})i}} \right]$$

Equation 14.15:

$$\mathrm{V}(b_p) = \frac{\sigma^2}{\sum_{i=1}^{n} e^2_{(x_p, x_1 \ldots x_{p-1} x_{p+1} \ldots x_k)i}}$$

Equation 14.16:

$$s^2 = \frac{\sum_{i=1}^{n} e_i^2}{n-k-1}$$

Equation 14.17:

$$\text{adjusted } R^2 = 1 - \frac{\left(\dfrac{\sum_{i=1}^{n} e_i^2}{n-k-1}\right)}{\left(\dfrac{\sum_{i=1}^{n} \left(y_i - \overline{y}\right)^2}{n-1}\right)} = 1 - \frac{s^2}{\mathrm{V}\left(y_i\right)}$$

Equation 14.18:

$$\mathrm{SD}\left(b_m\right) = +\sqrt{\frac{s^2}{\sum_{i=1}^{n} e^2_{\left(x_m, x_1 \ldots x_{m-1} x_{m+1} \ldots x_k\right)i}}} = +s\sqrt{\frac{1}{\sum_{i=1}^{n} e^2_{\left(x_m, x_1 \ldots x_{m-1} x_{m+1} \ldots x_k\right)i}}}$$

Equation 14.19:

$$\frac{b_p - \beta_p}{\sqrt{\dfrac{s^2}{\sum_{i=1}^{n} e^2_{\left(x_p, x_1 \ldots x_{p-1} x_{p+1} \ldots x_k\right)i}}}} \sim t^{(n-k-1)}$$

Equation 14.20:

$$\frac{b_m - \beta_m}{\sqrt{\dfrac{s^2}{\sum_{i=1}^{n} e^2_{(x_m, x_1 \ldots x_{m-1} x_{m+1} \ldots x_k) i}}}} \sim \mathrm{N}(0,1)$$

Equation 14.21:

$$\frac{\left(\dfrac{\left(\sum_{i=1}^{n} e_i^2 \right)_{\mathrm{R}} - \left(\sum_{i=1}^{n} e_i^2 \right)_{\mathrm{U}}}{j} \right)}{\left(\dfrac{\left(\sum_{i=1}^{n} e_i^2 \right)_{\mathrm{U}}}{n-k-1} \right)} \sim F\left(j, n-k-1 \right)$$

Equation 14.22:

$$\frac{R_{\mathrm{U}}^2}{1-R_{\mathrm{U}}^2} \frac{n-k-1}{k} \sim F\left(k, n-k-1 \right)$$

Equation 14.23:

$$y_i = \delta_0 + \sum_{l=2}^{k} \delta_l x_{li} + \varepsilon_i$$

Equation 14.24:

$$y_i = \mu_0 + \sum_{l=2}^{k} \mu_l x_{li} + \varepsilon_i$$

Equation 14.25:

$$y_i = \delta_1 (1 - x_{1i}) + \sum_{l=2}^{k} \delta_l (1 - x_{1i}) x_{li} + \mu_1 x_{1i} + \sum_{l=2}^{k} \mu_l x_{1i} x_{li} + \varepsilon_i$$

Equation 14.26:

$$\frac{\left(\frac{\left(\sum_{i=1}^{n} e_i^2 \right)_{\mathrm{R}} - \left(\left(\sum_{i=1}^{n} e_i^2 \right)_{x_{1i}=0} + \left(\sum_{i=1}^{n} e_i^2 \right)_{x_{1i}=1} \right)}{k-1} \right)}{\left(\frac{\left(\sum_{i=1}^{n} e_i^2 \right)_{x_{1i}=0} + \left(\sum_{i=1}^{n} e_i^2 \right)_{x_{1i}=1}}{n-2k} \right)} \sim F(k-1, n-2k)$$

Equation 14.27:

$$\frac{\left(\frac{1{,}512{,}607{,}000{,}000 - (1{,}087{,}613{,}000{,}000 + 405{,}560{,}200{,}000)}{8-1} \right)}{\left(\frac{(1{,}087{,}613{,}000{,}000 + 405{,}560{,}200{,}000)}{1{,}000 - 2(8)} \right)} \sim F(8-1, 1{,}000 - 2(8))$$

Equation 14.28:

Years of schooling	=	(No schooling completed)	×	0
		(Nursery school to 4th grade)	×	4
		(5th or 6th grade)	×	6
		(7th or 8th grade)	×	8
		(9th grade)	×	9
		(10th grade)	×	10
		(11th grade)	×	11
		(12th grade, no diploma)	×	11
		(High school graduate)	×	12
		(Some college, but less than one year)	×	12
		(One or more years of college, no degree)	×	13
		(Associate degree)	×	14
		(Bachelor’s degree)	×	16
		(Master’s degree)	×	18
		(Professional degree)	×	19
		(Doctorate degree)	×	21

Equation 14.29:

$$\frac{\left(\frac{318,767,600,000,000-298,963,700,000,000}{15}\right)}{\left(\frac{298,963,700,000,000}{179,549-24}\right)} \sim F\left(15,179,525\right)$$

Equation 14.30:

$$y_i = \alpha + \beta x_i + \gamma a_i + \varepsilon_i$$

Equation 14.31:

$$y_i = \alpha + \beta x_i + \left(\gamma a_1 d_1 + \gamma a_2 d_2 + \cdots + \gamma a_n d_n\right) + \varepsilon_i$$

Equation 14.32:

$$y_i = \alpha + \beta x_i + \left(\gamma a_1\right) d_1 + \left(\gamma a_2\right) d_2 + \cdots + \left(\gamma a_n\right) d_n + \varepsilon_i$$

Equation 14.33:

$$y_i = a + b_0 x_i + b_1 d_1 + b_2 d_2 + \cdots + b_n d_n + e_i$$

Equation 14.34:

$$\begin{aligned} y_{it} &= \alpha + \beta p x_{i1} + \left(p\gamma a_{11}\right) d_1 + \left(p\gamma a_{21}\right) d_2 + \cdots + \left(p\gamma a_{n-1,1}\right) d_{n-1} \\ &+ \beta\left(1-p\right) x_{i2} + \left(\left(1-p\right)\gamma a_{12}\right) d_1 + \left(\left(1-p\right)\gamma a_{22}\right) d_2 + \cdots + \left(\left(1-p\right)\gamma a_{n2}\right) d_n + \varepsilon_{it} \end{aligned}$$

Equation 14.35:

$$y_{i1} = \alpha + \beta x_{i1} + \gamma a_i + \varepsilon_{i1}$$

Equation 14.36:

$$y_{i2} = \alpha + \beta x_{i2} + \gamma a_i + \varepsilon_{i2}$$

Equation 14.37:

$$\left(y_{i2}-y_{i1}\right)=\beta\left(x_{i2}-x_{i1}\right)+\left(\varepsilon_{i2}-\varepsilon_{i1}\right)$$